

# 时间序列期末答案

September 2020

## 一、 填空题

1.  $ARIMA(0, 1, 1)$
2.  $4, ARIMA(0, 1, 1) \times (1, 1, 1)_4$
3.  $[\hat{\gamma}_1 - 0.098\hat{\gamma}_0, \hat{\gamma}_1 + 0.098\hat{\gamma}_0]$
4.  $2 \cos \omega X_3 - X_2$
5. 5.75
6.  $\alpha_0 + \alpha_1 \epsilon_{t-1}^2$
7.  $\frac{8}{21}, \frac{1}{21}$
8. 用  $\{X_t, t \leq n\}$  对  $X_{n+1}$  做线性预测的误差非 0,  $\lim_{k \rightarrow \infty} \sigma_k = \gamma_0$
9.  $(1 + \theta_1^2 + \cdots + \theta_{l-1}^2)\sigma^2$
10. 0

二、

$$\begin{aligned}X_t &= \frac{1+0.8\mathcal{B}}{1-0.4\mathcal{B}}\epsilon_t \\&= (1+0.8\mathcal{B})\sum_{i=0}^{\infty}0.4^i\mathcal{B}^i\epsilon_t \\&= \epsilon_t + 3\sum_{i=1}^{\infty}0.4^i\epsilon_{t-i}\end{aligned}$$

故有  $Var(X_t) = [1 + 9\sum_{i=1}^{\infty}0.4^{2i}]\sigma^2 = \frac{19}{7}\sigma^2$

$$Cov(X_{t+k}, \epsilon_t) = \begin{cases} \sigma^2 & k=0 \\ 3 \times 0.4^k \sigma^2 & k>0 \\ 0 & k<0 \end{cases}$$

$Var(\epsilon_t) = \sigma^2$ , 故

$$\rho_{X,\epsilon}(k) = \begin{cases} 0 & k<0 \\ \sqrt{\frac{7}{19}} & k=0 \\ \sqrt{\frac{7}{19}} \times 3 \times 0.4^k & k>0 \end{cases}$$

有些同学把根号化成了小数, 也被认为是正确的。

三、

讨论,  $l=2k$  时,  $\hat{X}_t(l) = \frac{1}{9^k}X_t$ ,  $l=2k-1$  时,  $\hat{X}_t(l) = \frac{1}{9^k}X_{t-1}$ .  
对应的预测方差为  $Var(\hat{X}_t(l) - X_{t+l}) = (1 + \frac{1}{9^{2k}})\gamma_0 - \frac{2}{9^k}\gamma_{2k}$ .  
由此易知预测方差趋于  $\gamma_0$ .

四、 参见 1.2 习题课讲义

参见 2019 年期末试卷, 答案为 0.2836, 1.018, 是正态分布

五、

1、

$$(1 - \phi_1 \mathcal{B})X_t = \phi_0 + (1 - \theta_1 \mathcal{B})\epsilon_t$$

$$\begin{aligned} X_t &= \frac{\phi_0}{1 - \phi_1 \mathcal{B}} + \frac{1 - \theta_1 \mathcal{B}}{1 - \phi_1 \mathcal{B}} \epsilon_t \\ &= \frac{\phi_0}{1 - \phi_1} + (1 - \theta_1 \mathcal{B}) \sum_{i=0}^{\infty} \phi_1^i \mathcal{B}^i \epsilon_t \\ &= \frac{\phi_0}{1 - \phi_1} + (1 - \theta_1 \mathcal{B}) \sum_{i=0}^{\infty} \phi_1^i \epsilon_{t-i} \\ &= \frac{\phi_0}{1 - \phi_1} + \sum_{i=1}^{\infty} \phi_1^i \epsilon_{t-i} - \theta_1 \sum_{i=1}^{\infty} \phi_1^i \epsilon_{t-i-1} \\ &= \frac{\phi_0}{1 - \phi_1} + \epsilon_t + \left(1 - \frac{\theta_1}{\phi_1}\right) \sum_{i=1}^{\infty} \phi_1^i \epsilon_{t-i} \end{aligned}$$

2、

$$\begin{aligned} \epsilon_t &= \frac{1 - \phi_1 \mathcal{B}}{1 - \theta_1 \mathcal{B}} X_t - \frac{\phi_0}{1 - \theta_1 \mathcal{B}} \\ &= -\frac{\phi_0}{1 - \theta_1} + X_t + \left(1 - \frac{\phi_1}{\theta_1}\right) \sum_{i=1}^{\infty} \theta_1^i X_{t-i} \end{aligned}$$

3、

对 1. 中结果求期望, 得  $E(X_t) = \frac{\phi_0}{1 - \phi_1}$

$k = 0$  时:

$$\begin{aligned} Cov(X_t, X_t) &= \left[ 1 + \left(1 - \frac{\theta_1}{\phi_1}\right)^2 \frac{\phi_1^2}{1 - \phi_1^2} \right] \sigma^2 \\ &= \left[ 1 + \frac{(\phi_1 - \theta_1)^2}{1 - \phi_1^2} \right] \sigma^2 \end{aligned}$$

$k > 0$  时:

$$\begin{aligned} Cov(X_t, X_{t+k}) &= \left[ \left(1 - \frac{\theta_1}{\phi_1}\right) \phi_1^k + \left(1 - \frac{\theta_1}{\phi_1}\right)^2 \frac{\theta_1^{k+2}}{1 - \phi_1^2} \right] \sigma^2 \\ &= \left[ \left(1 - \frac{\theta_1}{\phi_1}\right) + \frac{(\theta_1 - \phi_1)^2}{1 - \phi_1^2} \right] \phi_1^k \sigma^2 \end{aligned}$$