1. 计算 $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}_{2021}, \mathbb{Z}_{86})$, $\mathbb{Z}_{2021} \otimes_{\mathbb{Z}} \mathbb{Z}_{2009}$

2. M f.g A-module. M Noetherian A-module $\iff$ $A/\mathrm{Ann}(M)$ Noetherian ring

3. $P \in \mathrm{Spec}(\mathcal{O}_K)$, $P \neq 0$. 证明 $P = (p, \alpha)$, $\underset{\text{prime}}{p \in \mathbb{Z}}$, $\alpha \in \mathcal{O}_K$

4. $\mathrm{Spec}\, \mathbb{Z}[i]$

5. A Noetherian $\iff$ $\forall p \in \mathrm{Spec} A$ f.g.

6. (1) $k[x, y]$ integrally closed

(2) $(x^2 - y^3)$ prime

(3) Is $A = k[x, y]/(x^2 - y^3) = k[\bar{x}, \bar{y}]$ integrally closed?

(4) minimal ~~primary~~ primary decomposition of $(\bar{y} - 1)$ of A

7. $A = \mathbb{Z}[\sqrt{-5}]$, $I = (2, 1+\sqrt{-5})$

(1) P prime, $2 \in P$. 证 $P = I$, $I_P = (1+\sqrt{-5})A_P$

(2) Is $(5, 1+\sqrt{-5})$ primary?

Is it a power of a prime ideal?

(3) prime ideal decompositions of (2), (3), (5) in A

8. A Dedekind domain. $S^{-1}A$ is either Dedekind domain or ~~field~~ field.

9. A DVR. $x, y \in K$, $v(x) \neq v(y)$ 证 $v(x+y) = \min\{v(x), v(y)\}$

10. $\underset{\text{integral domain}}{B \subset A}$ A is integral over B. then $\dim B = \dim A$

Add: noetherian integral domain A is UFD $\iff$ every prime ideal of height 1 is principal.